

# CIS 4360, SPRING 2026

## HASH FUNCTION

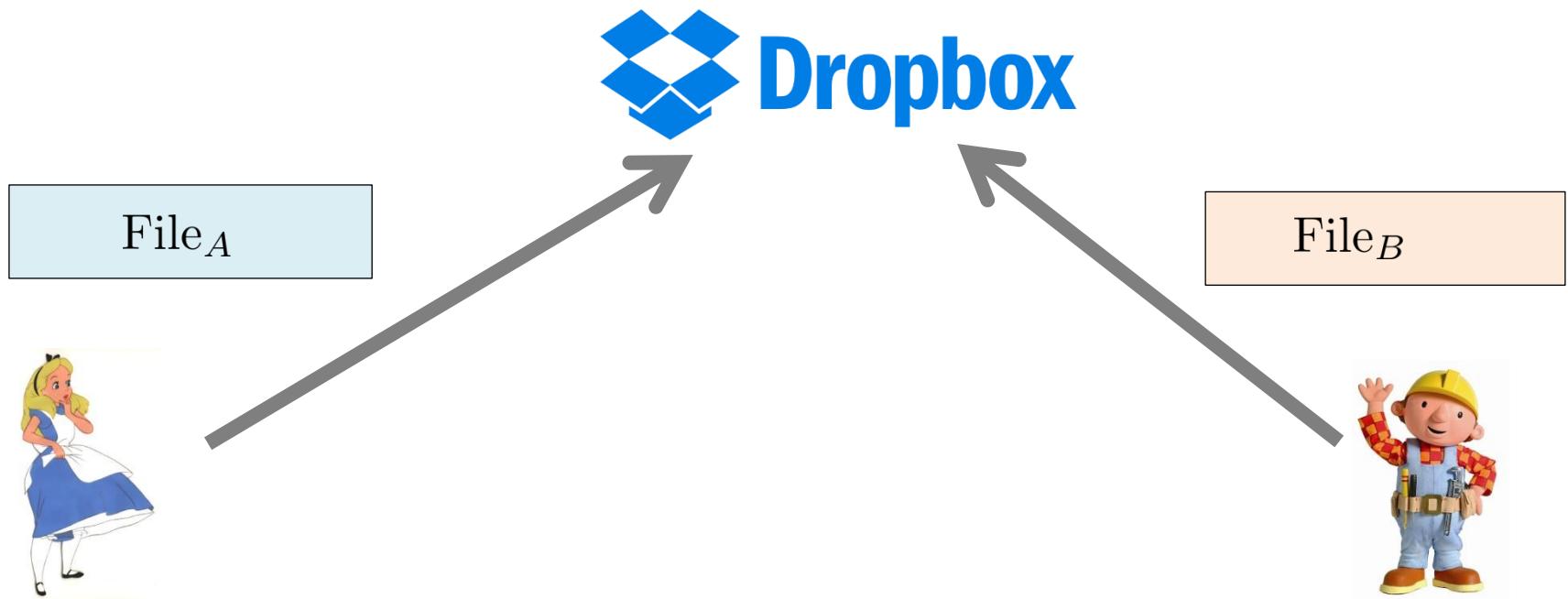
VIET TUNG HOANG

# Agenda

---

- 1. Security Modeling for Hash Functions**
2. Building Hash Function: MD Transform
3. Application: Password Storage

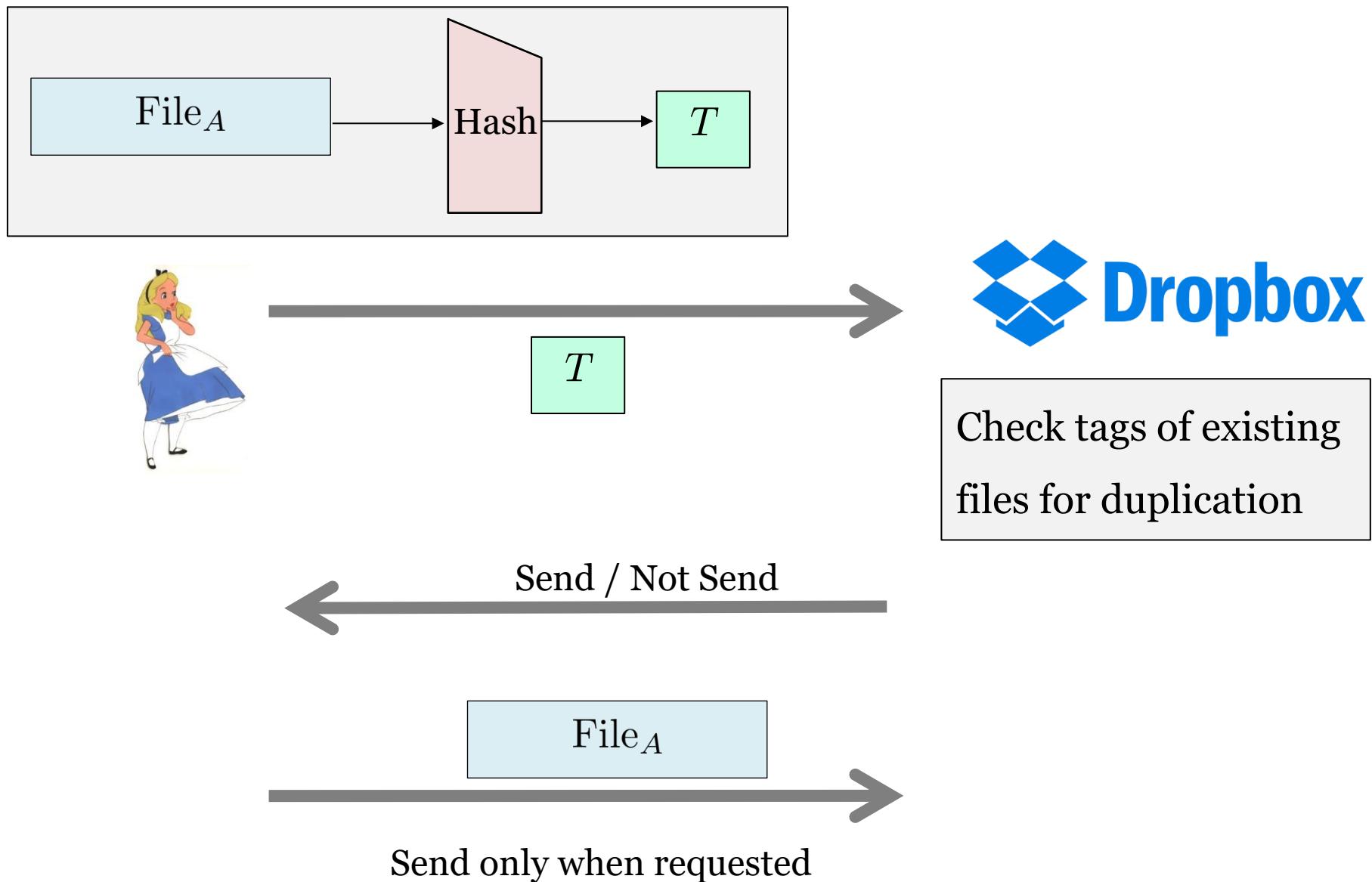
# Motivating Application: Data Deduplication



## Dropbox's goals:

- If many users store the same file, keep only a **single** copy
- Minimize bandwidth usage

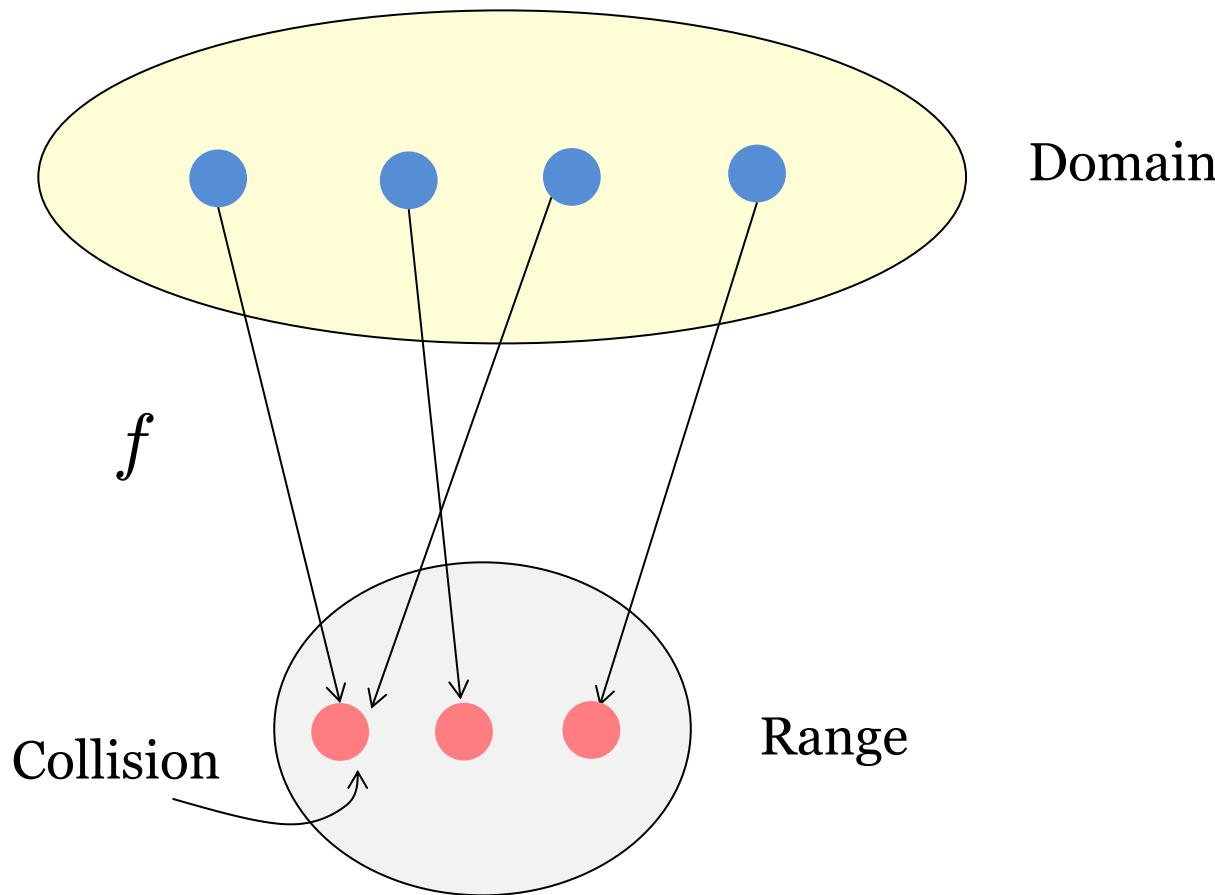
# Motivating Application: Data Deduplication



# What property do we need for the hash?

# Collision-Resistance

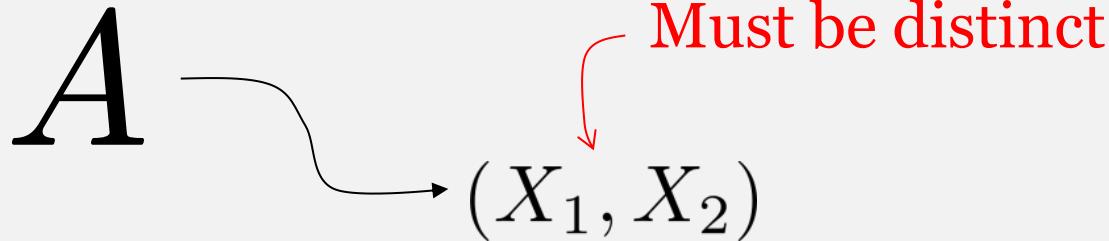
$f : \text{Domain} \rightarrow \text{Range}$



By Pigeonhole Principle, if  $|\text{Domain}| > |\text{Range}|$  then collision exists

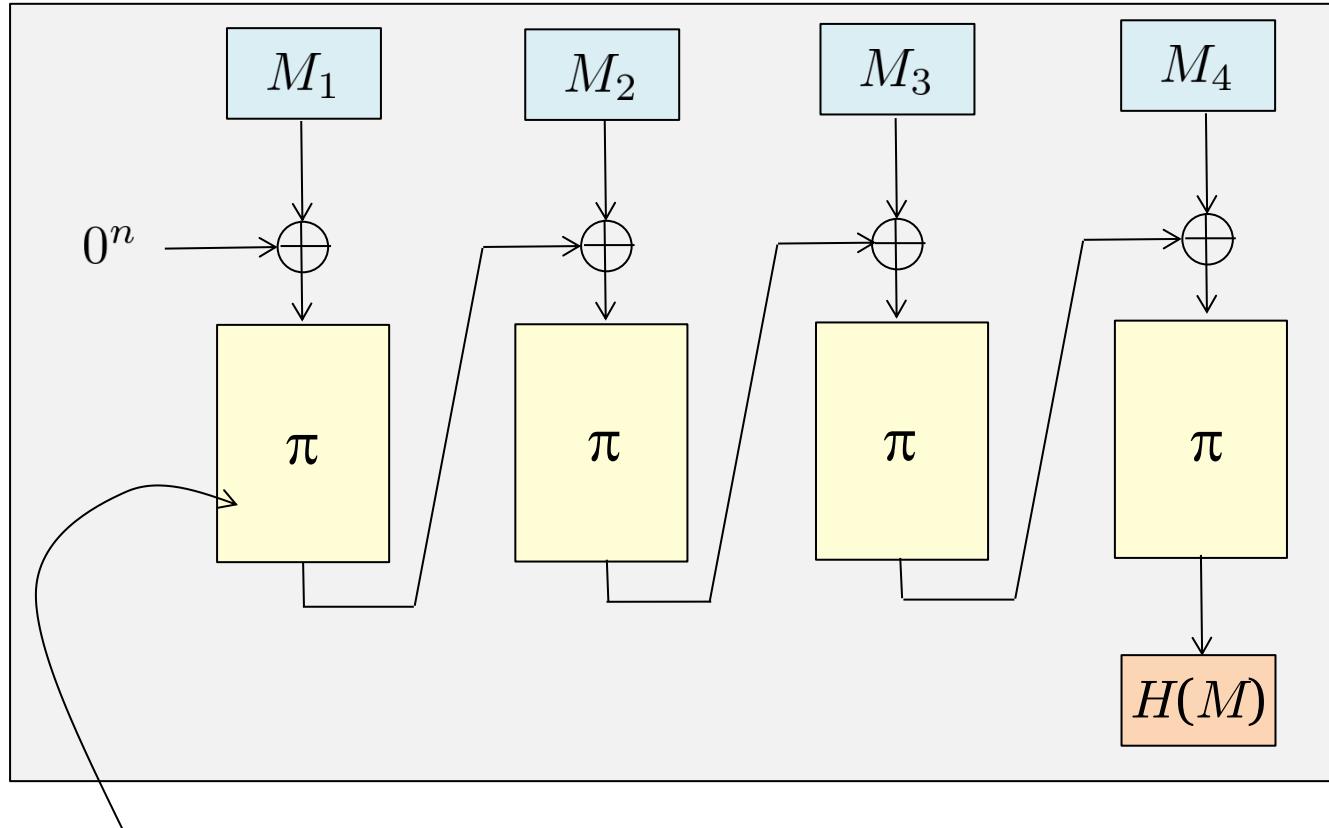
**Want:** collisions are hard to find, although they exist

# Defining Collision-Resistance



$$\mathbf{Adv}_H^{\text{cr}}(A) = \Pr[H(X_1) = H(X_2)]$$

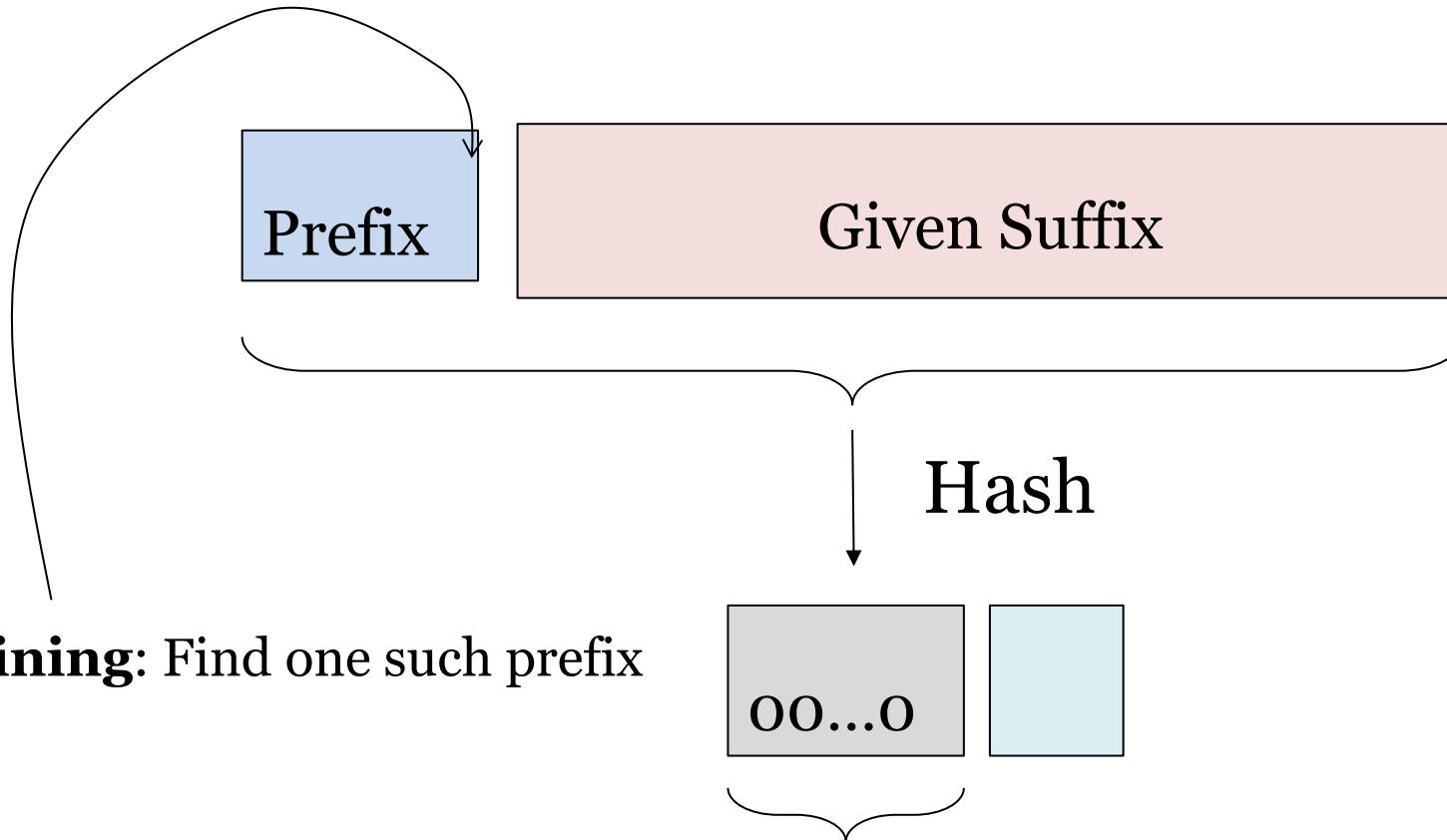
# Exercise: Break Collision Resistance



Public permutation

$\pi, \pi^{-1}$  are public

# CR Is Not Enough: Bitcoin Mining



**Mining:** Find one such prefix

**Want:** Can't mine faster than brute-force

# Modeling Security of Hash Functions

## The Random Oracle Model

RO



Maintain a secret random  $f : \{0, 1\}^n \rightarrow \{0, 1\}^m$

Everybody, including the adversary, has access to RO

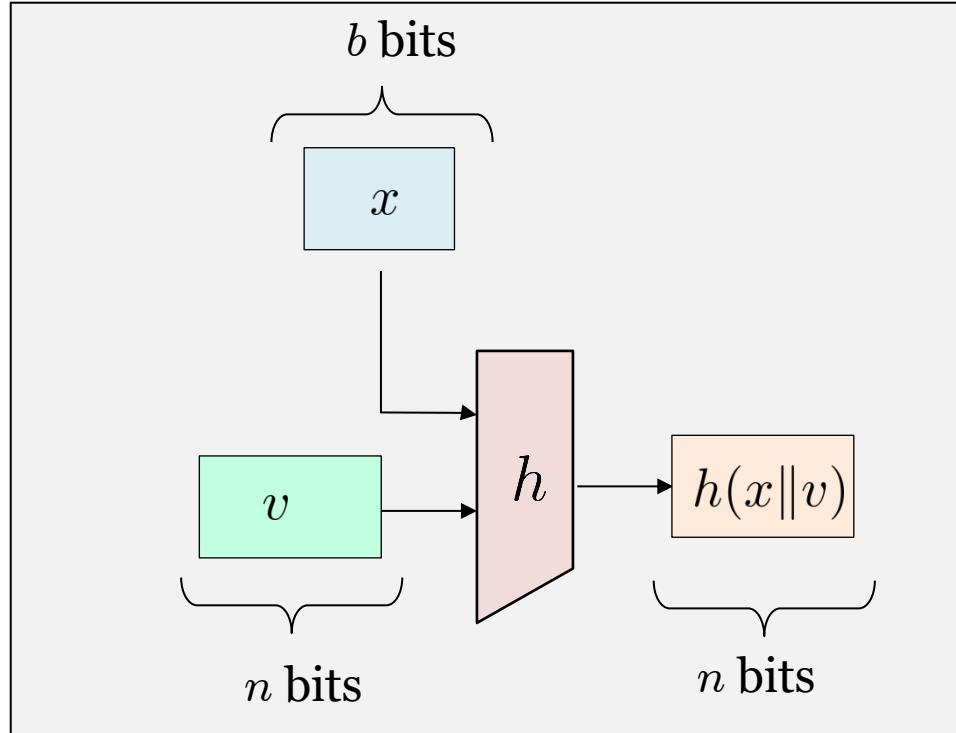
# Agenda

---

1. Security Modeling for Hash Functions
2. **Building Hash Function: MD Transform**
3. Application: Password Storage

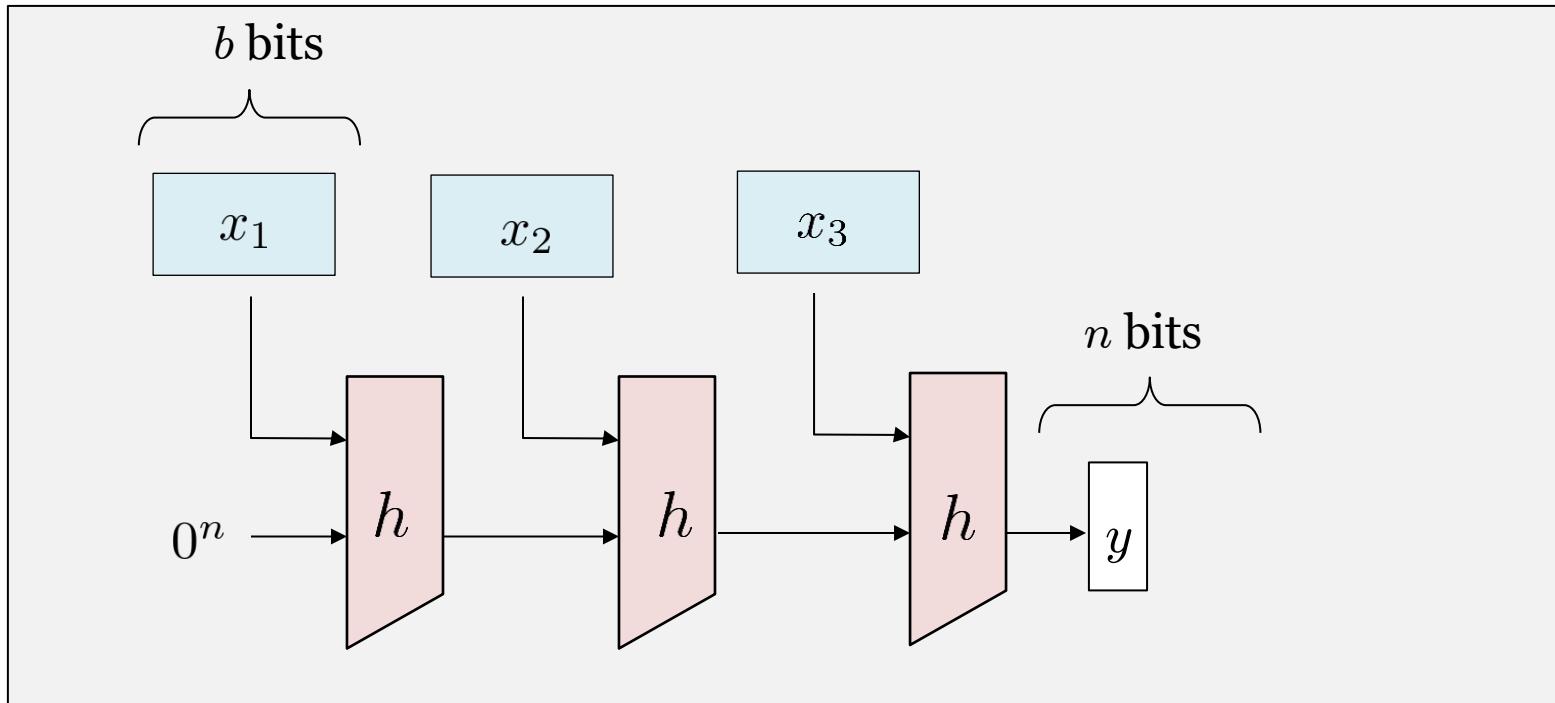
# Compression Functions

$$h : \{0, 1\}^{b+n} \rightarrow \{0, 1\}^n$$



For SHA-2,  $b = 512$  and  $n = 256$

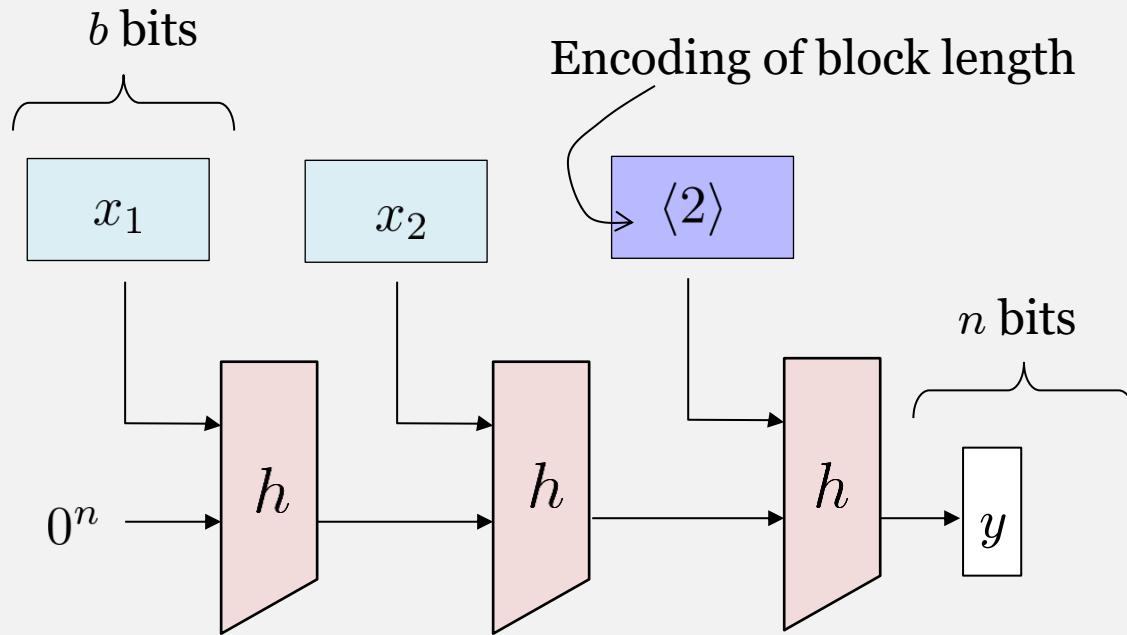
# First Attempt



**Question:** Suppose that  $h(0^b \parallel 0^n) = 0^n$

Break the collision resistance of  $H$

# Second Attempt: Plain Merkle-Damgard



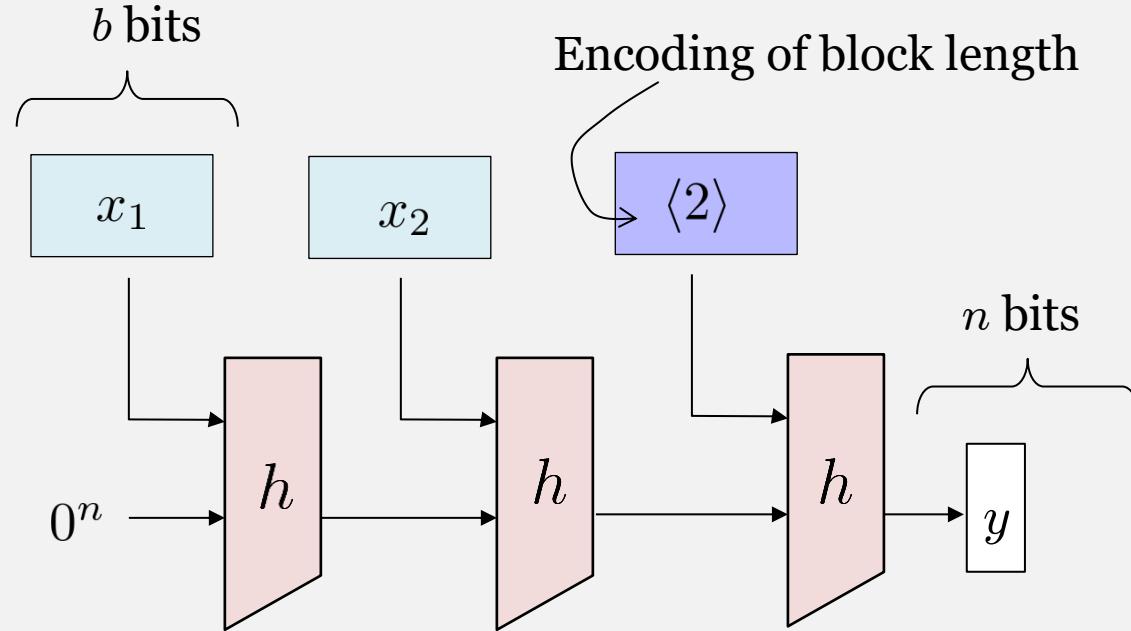
This is the structure of SHA-256

**Theorem:** If  $h$  is CR then  $H = \mathbf{MD}(h)$  is also CR

Can't attack  $H$  if  $h$  has no weakness

# Plain MD Is Not Enough for All Applications

## Length-Extension Attack



**Question:** Consider the following MAC  $F$

$$F_K(x) = H(K\|x)$$

Break the MAC security of  $F$  using a single Tag query

# The Damage of Length Extension Attack

## Hacking Trick: Bypass Authentication



$K$



bank.com

$K$

bank.com/api?token=ad6613c382&user=alice&cmd=NoOp

$H(K \parallel \text{"user=alice&cmd=NoOp"})$

Adversary tricks Alice to perform a harmless command to learn an authentication token



# The Damage of Length Extension Attack

## Hacking Trick: Bypass Authentication



$K$



bank.com

$K$

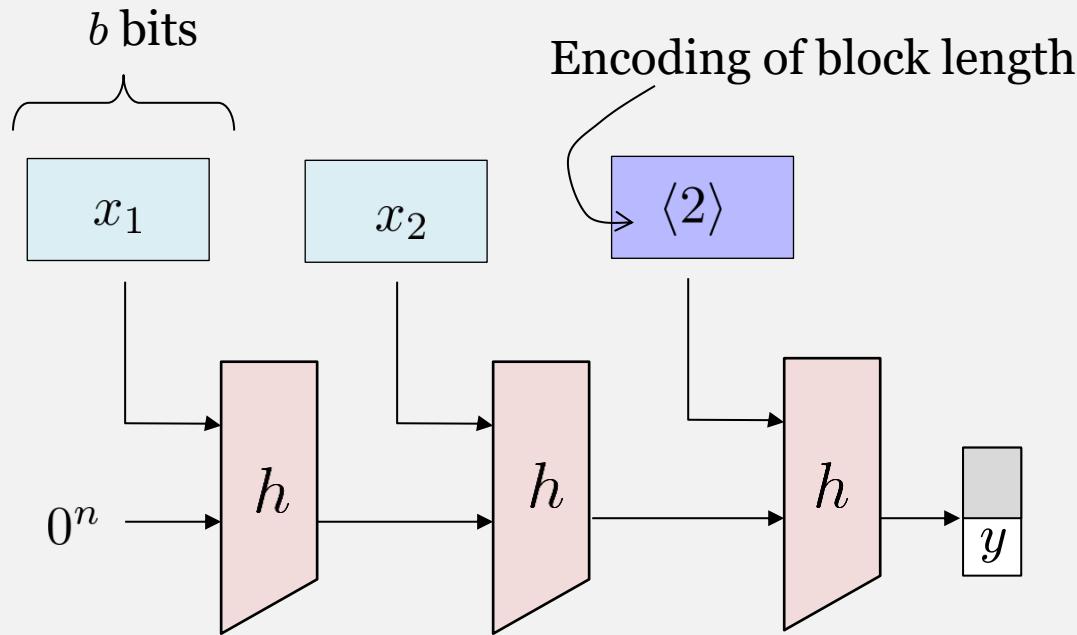
bank.com/api?token=dbb78b593f&user=alice&cmd=NoOp&cmd=OpenSafe

$H(K \parallel \text{"user=alice&cmd=NoOp&cmd=OpenSafe"})$

Adversary can compute the authentication token for a damaging command

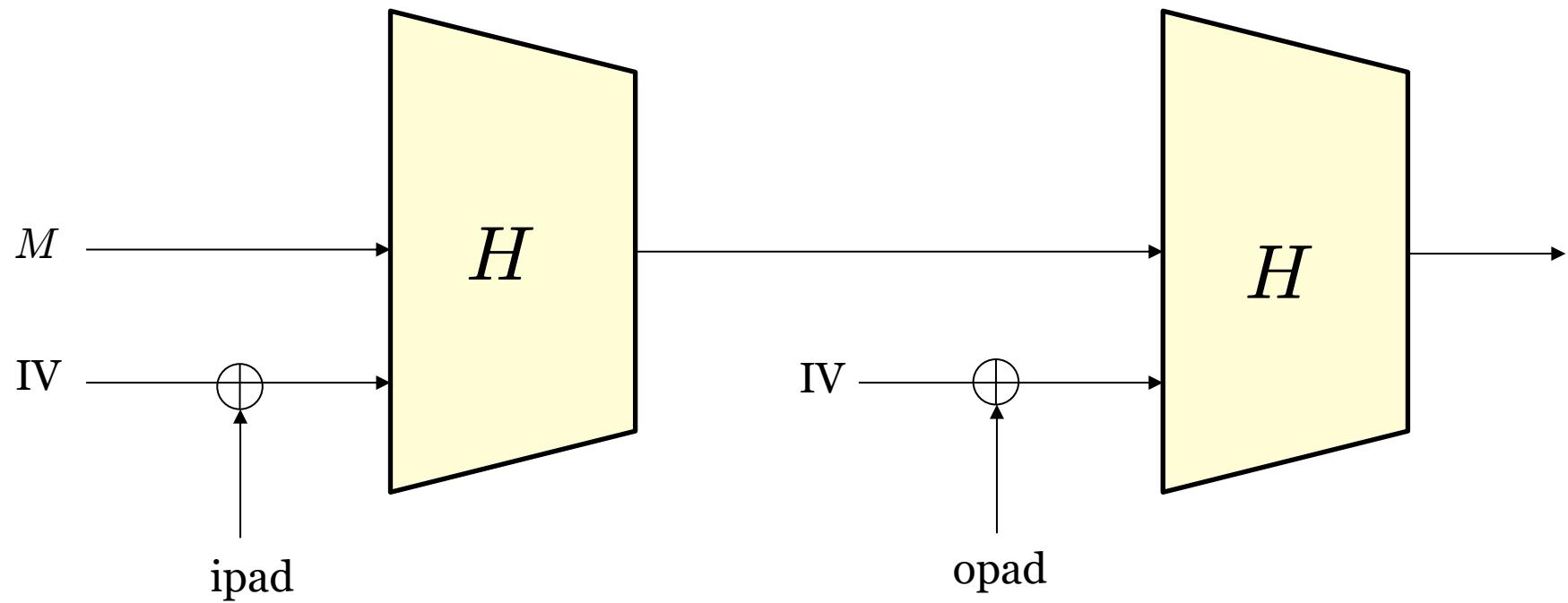


# The (Strengthened) MD Transform



The output needs to be truncated

# How To Have Large Output: HMAC



On large input, HMAC is only a bit more expensive than SHA-256

# Agenda

---

1. Security Modeling for Hash Functions
2. Building Hash Function: MD Transform
- 3. Application: Password Storage**

# Password Storage

**MOTHERBOARD**  
TECH BY VICE

## T-Mobile Stores Part of Customers' Passwords In Plaintext, Says It Has 'Amazingly Good' Security

A T-Mobile Austria customer representative made a surprising admission in a Twitter thread.

ars

TECHNICA

BIZ & IT —

### How an epic blunder by Adobe could strengthen hand of password crackers

Engineers flout universal taboo by *encrypting* 130 million pilfered passwords.

NEWS

## Hackers crack more than 60% of breached LinkedIn passwords

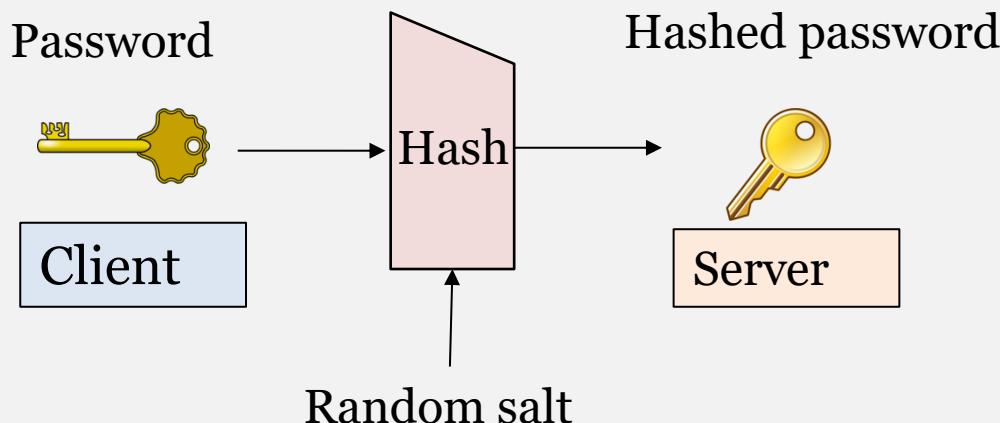
Speed of hackers to crack passwords shows weakness of security scheme used by LinkedIn, researchers say

# How Should Servers Store Users' Passwords?

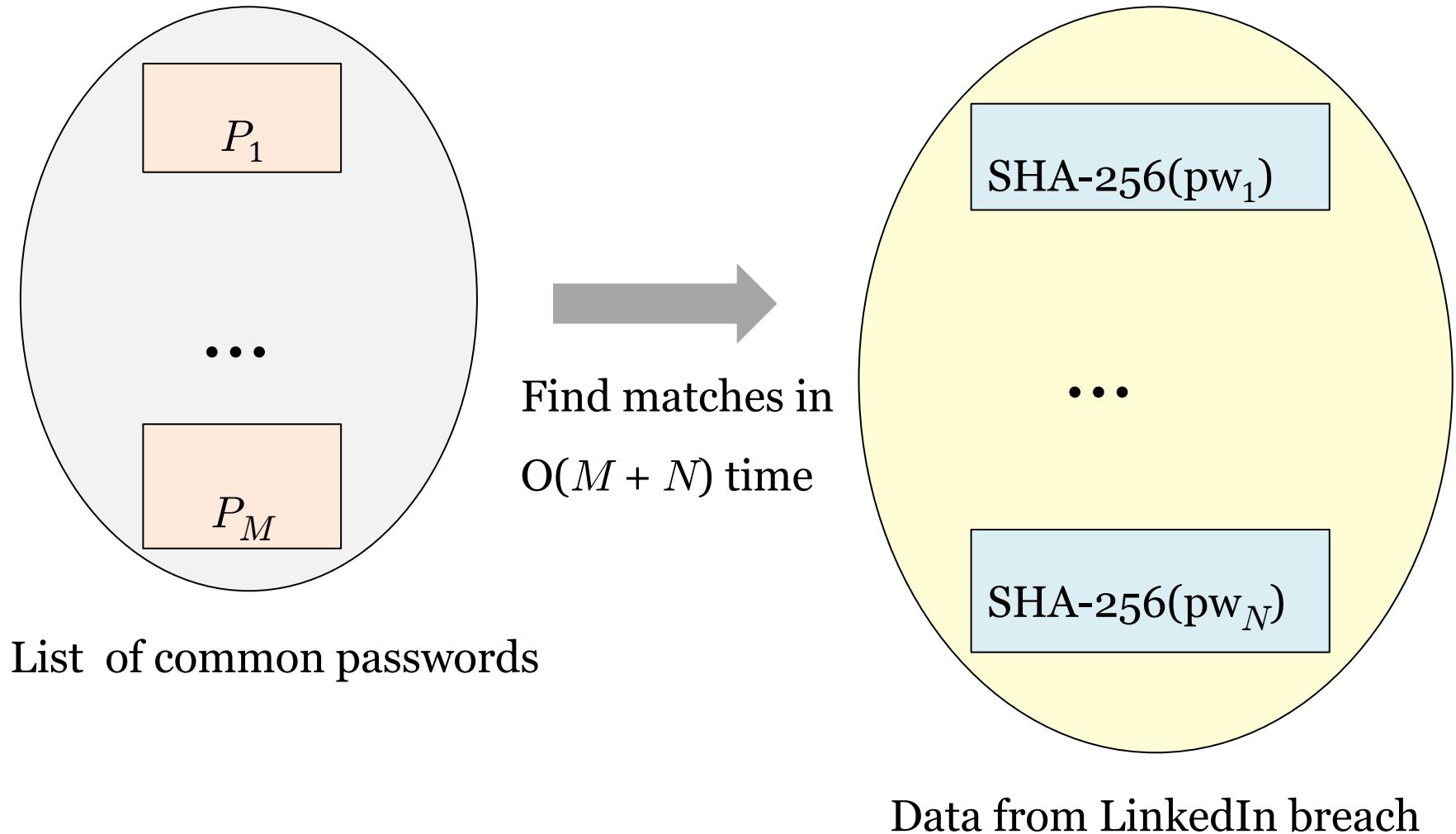
**Rule 1:** Only store hash outputs of passwords

Even server can't recover the passwords

**Rule 2:** Use a random salt for each user

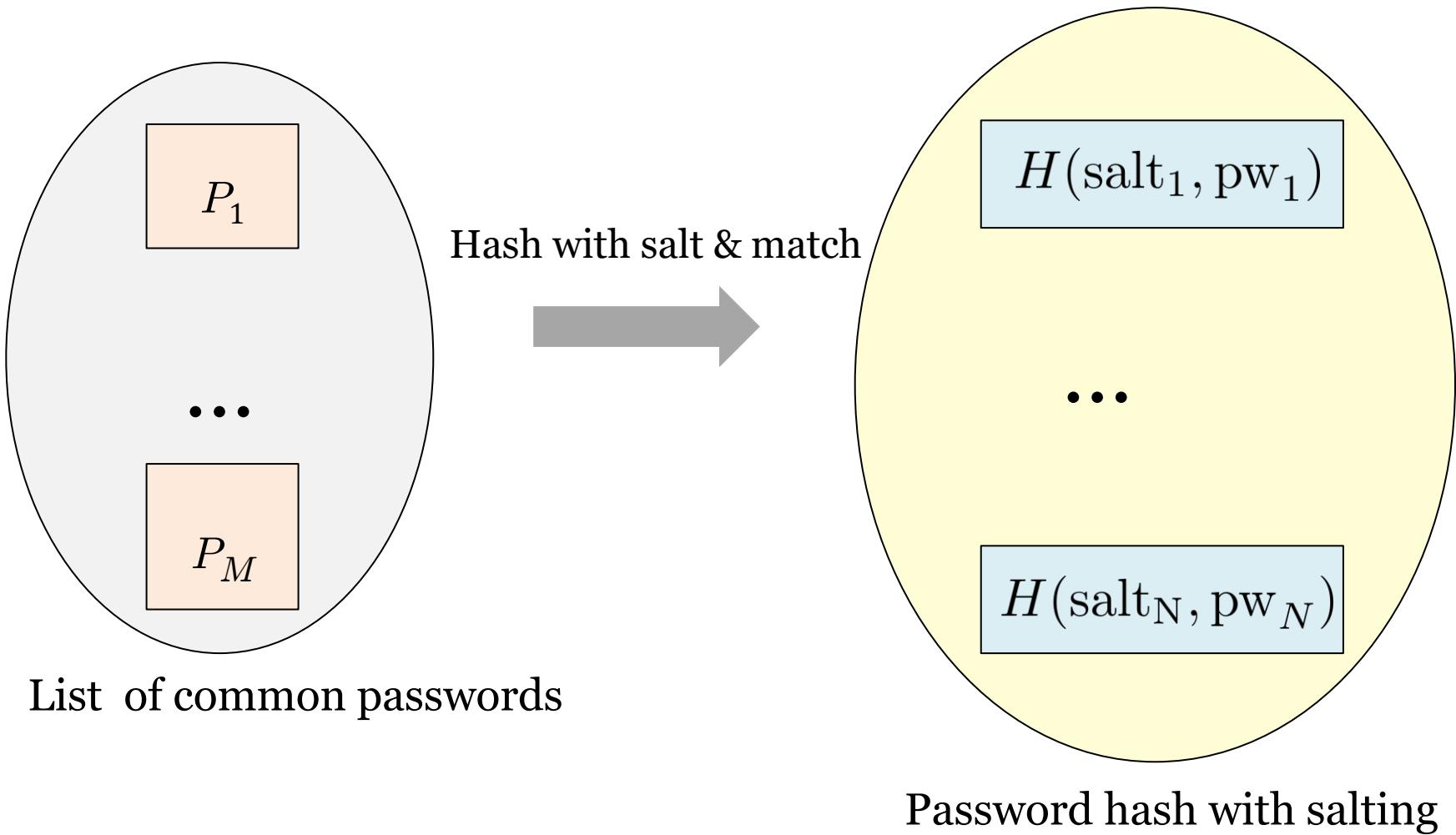


# Why Salts: Dictionary Attacks



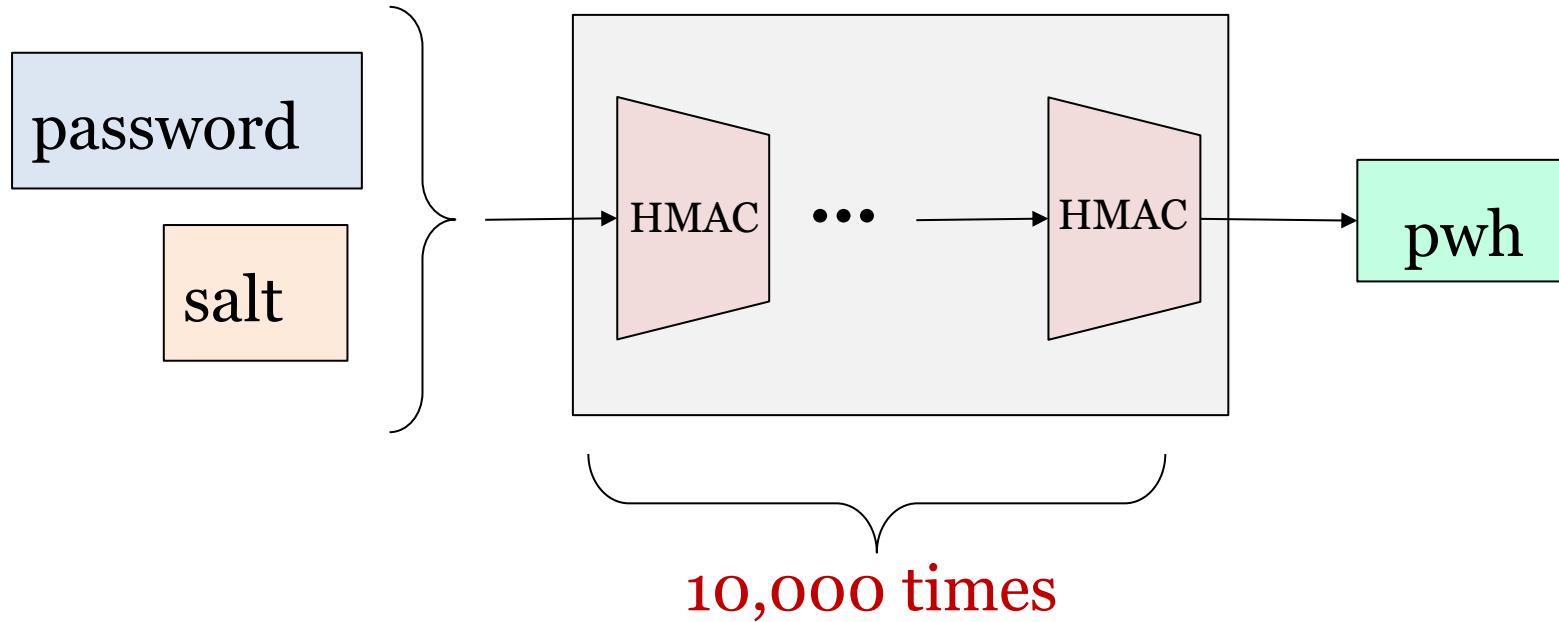
# Cost of Dictionary Attacks on Salting

Need  $\Theta(Mq)$  calls to  $H$  to recover  $q$  passwords



# Make It Even More Expensive

## Deliberately Slow Hashing



- Makes no difference for human users.
- Increase the cost of attackers for 10,000 times