CIS 5371, Fall 2025

Pseudorandom Function

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The slides are loosely based on those of Prof. Mihir Bellare, UC San Diego.

Agenda

1. Defining PRF Security

2. Birthday Attack

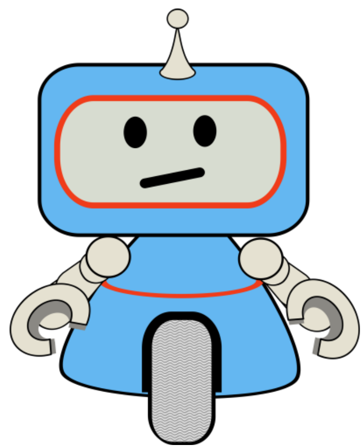
Recall

Possible Properties	**Necessary**	**Sufficient**
Security against key recovery	Yes	No
Hard to find M given $C \leftarrow E_K(M)$	Yes	No
…		

Want: a single "master" property that is sufficient to ensure security of common usage of blockcipher.

An Analogy: Turing Test

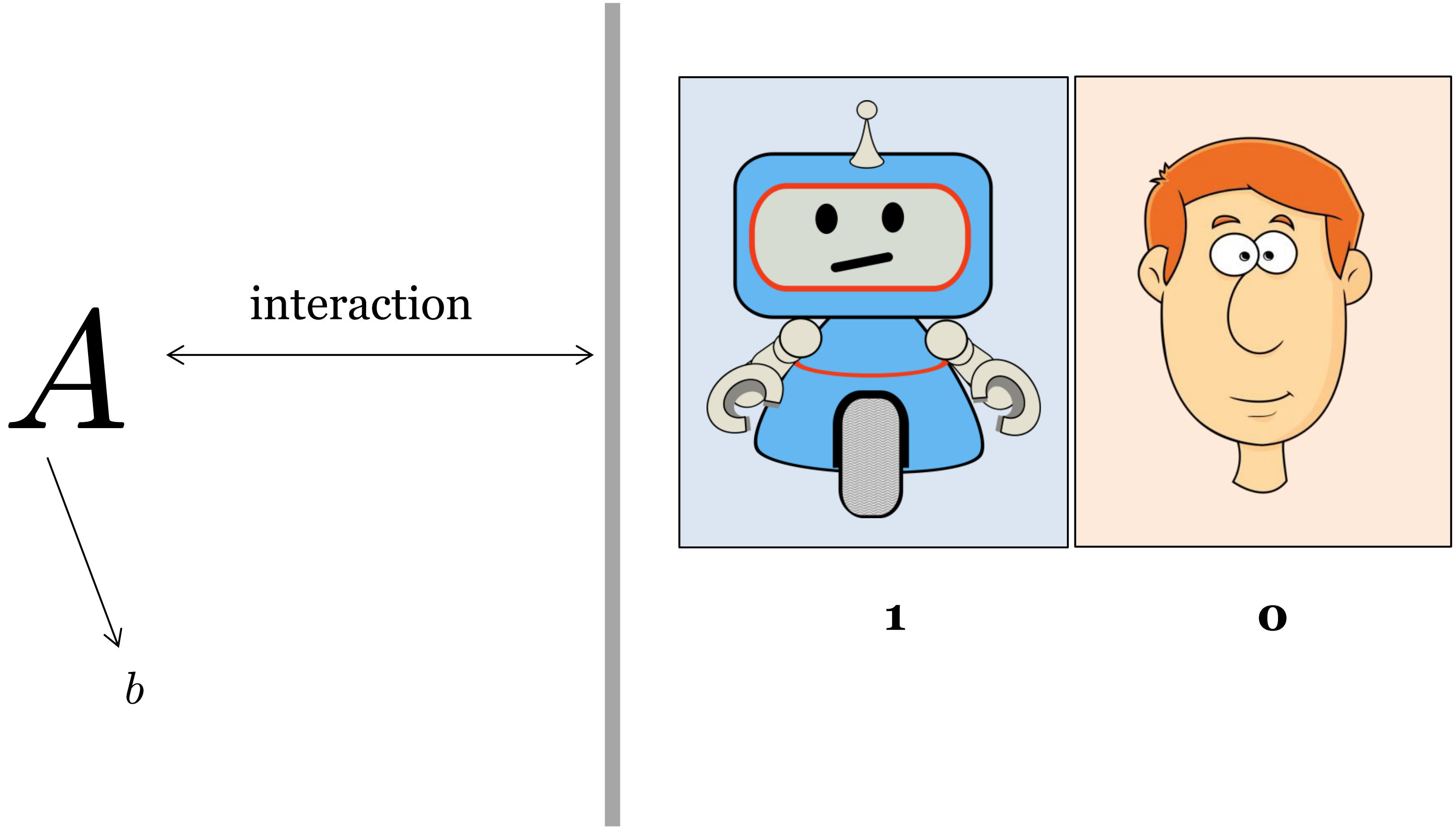
What does it mean for a machine to be “intelligent”?



Possible Answers
It can be happy
It recognizes pictures
…

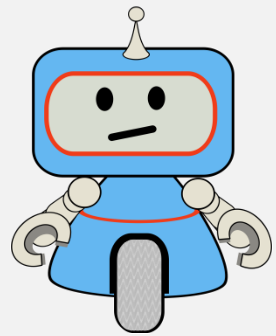
But no such list is satisfactory

An Analogy: Turing Test



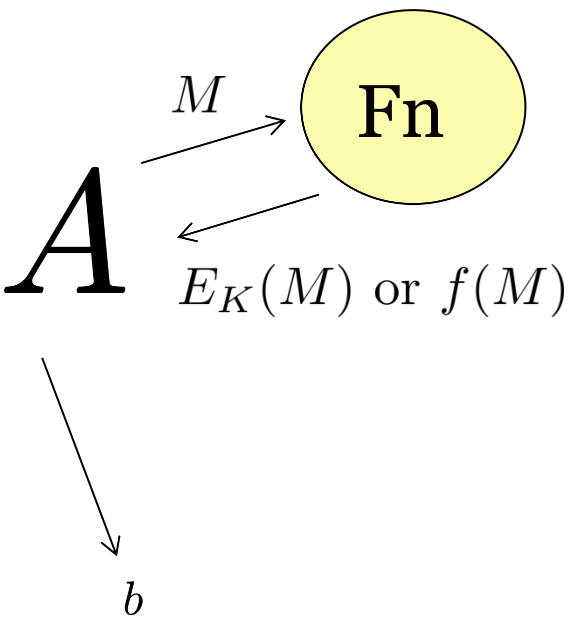
Man (0) or Machine (1)?

Real versus Ideal

Notion	Real object	Ideal object
Intelligence		
PRF	E_K	Random function

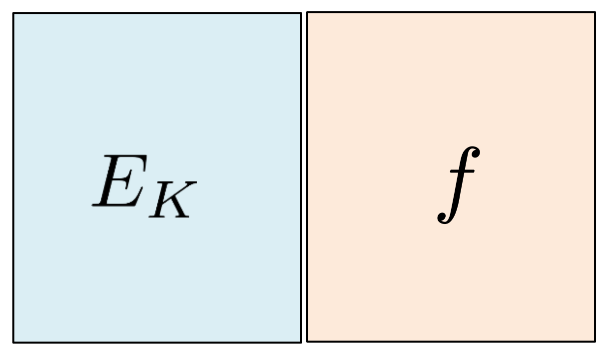
Informal View of PRF Security

$$E : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$$



Sample random $f : \{0,1\}^n \rightarrow \{0,1\}^n$

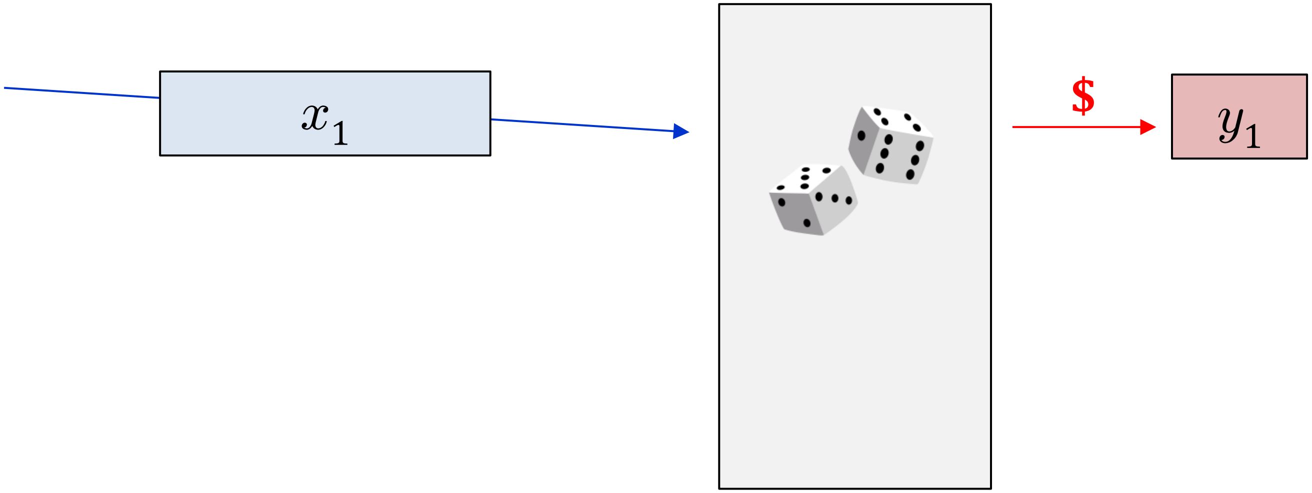
$K \xleftarrow{\$} \mathcal{K}$



Adversary doesn't know K or f

Defining Random Function: Lazy Sampling

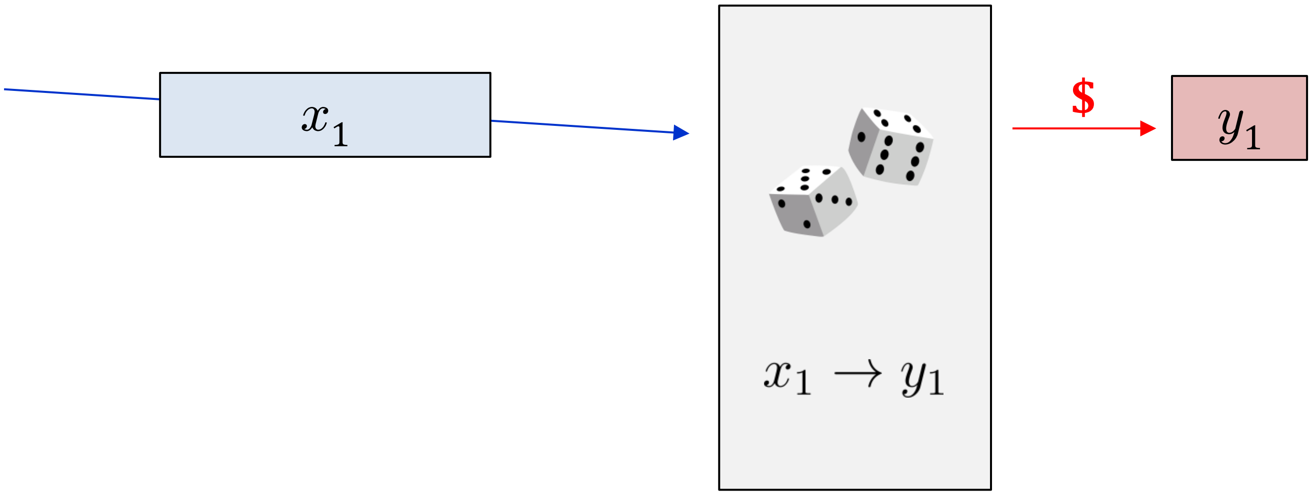
Want: a random function $f : \{0,1\}^n \to \{0,1\}^m$



Pick a fresh random answer for a new query, and remember the answer

Defining Random Function: Lazy Sampling

Want: a random function $f : \{0,1\}^n \to \{0,1\}^m$



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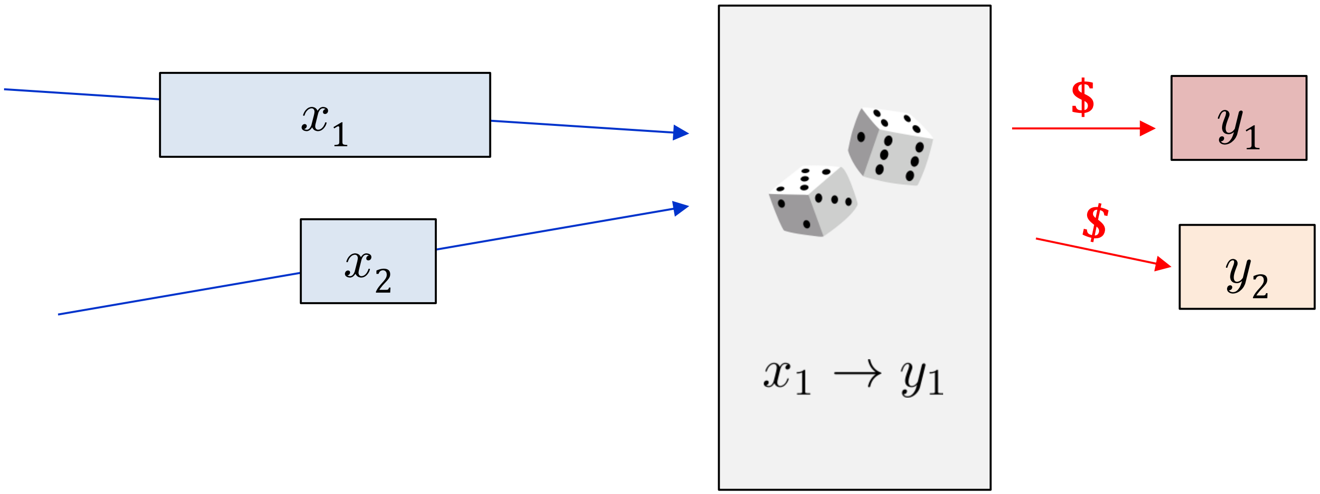

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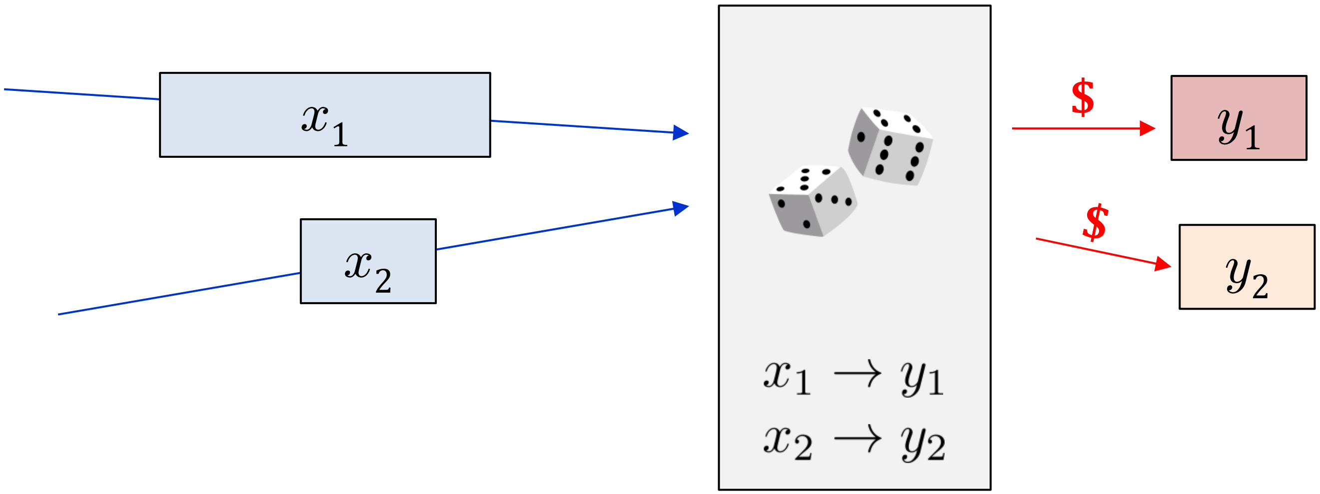
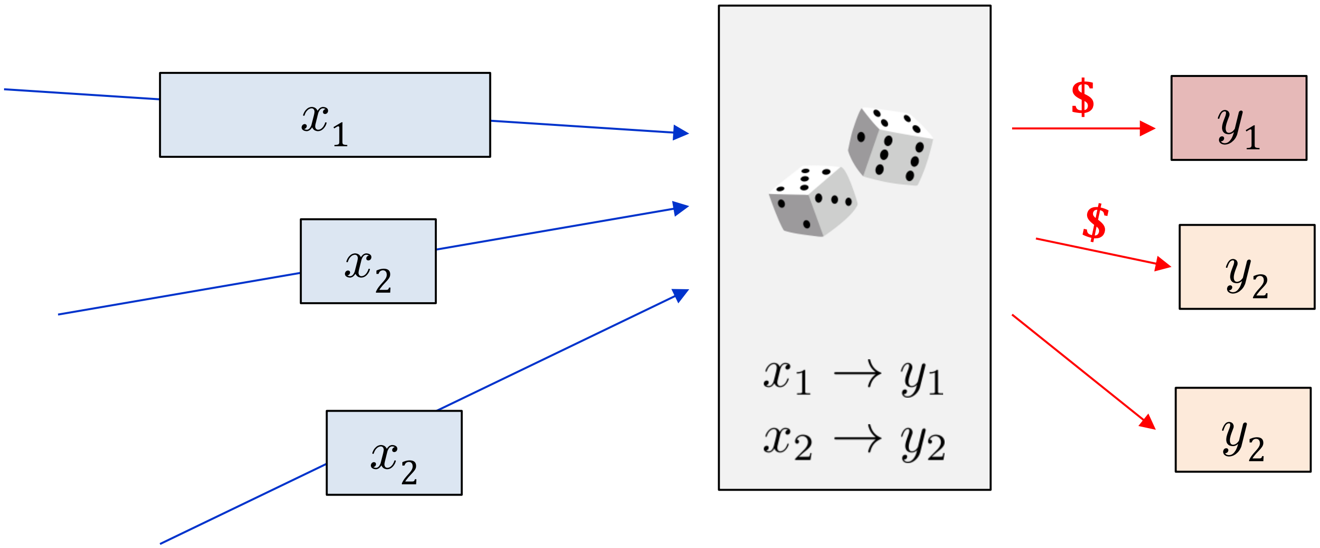


Pick a fresh random answer for a new query, and remember the answer

Reuse **Prior Answer** for Old Query

Want: a random function $f : \{0,1\}^n \to \{0,1\}^m$



Putting Things in Code

Game Real_E

procedure Initialize()

$K \xleftarrow{\$} \mathcal{K}$

procedure Fn(M)

return $E_K(M)$

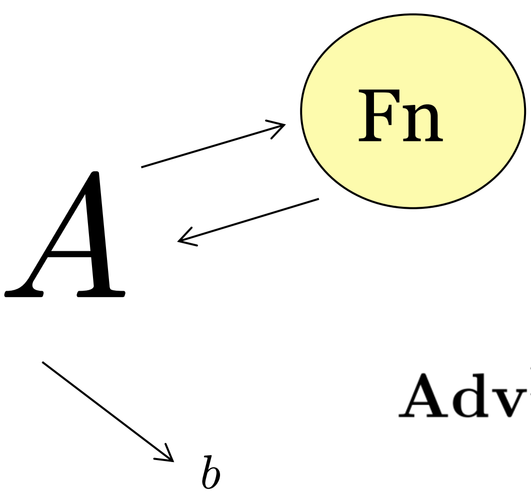
Game Rand_E

string array $T = \{\}$ // Global variable

procedure Fn(M)

If $T[M] = \perp$ then $T[M] \xleftarrow{\$} \{0,1\}^n$

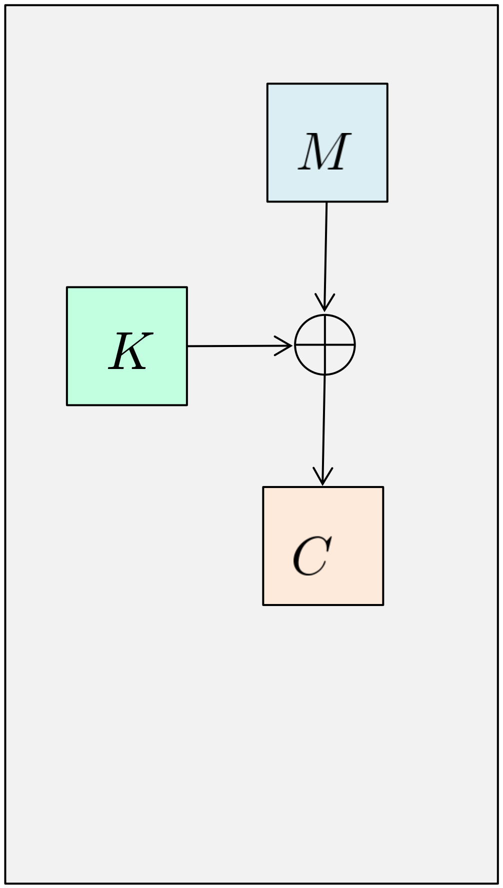
return $T[M]$



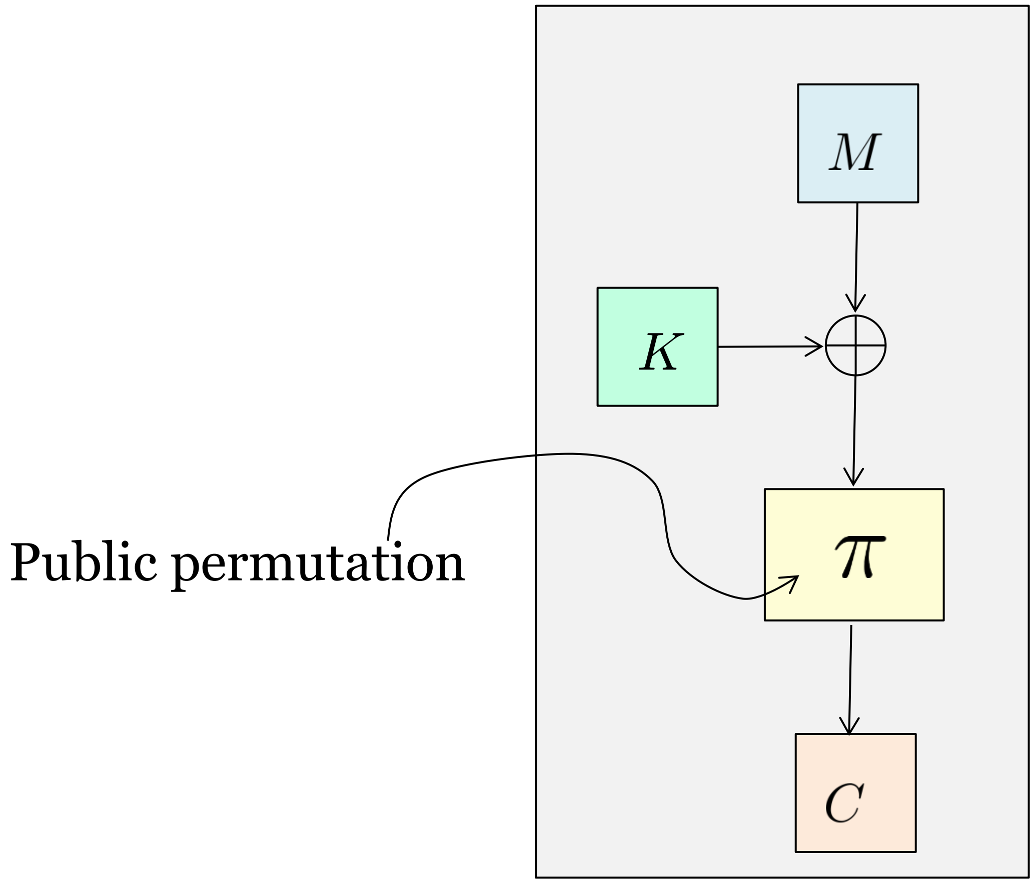
$$\mathbf{Adv}_E^{\mathrm{prf}}(A) = \Pr[\mathrm{Real}_E^A \Rightarrow 1] - \Pr[\mathrm{Rand}_E^A \Rightarrow 1]$$

Practice: PRF Attacks

$$E_K(M) = M \oplus K$$

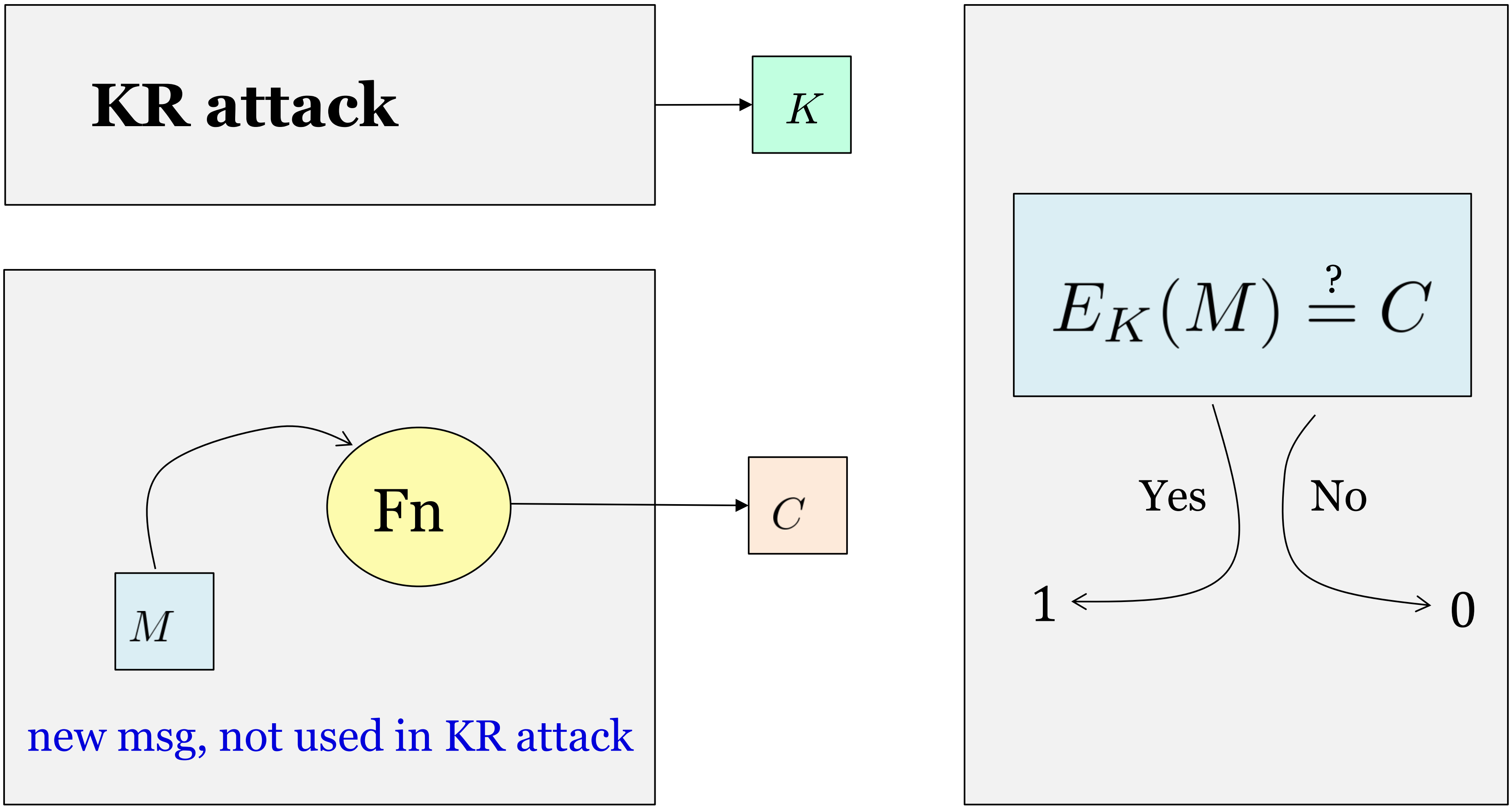


$$E_K(M) = \pi(M \oplus K)$$



π, π^{-1} are public

Easy to Break PRF Security After Key Recovery



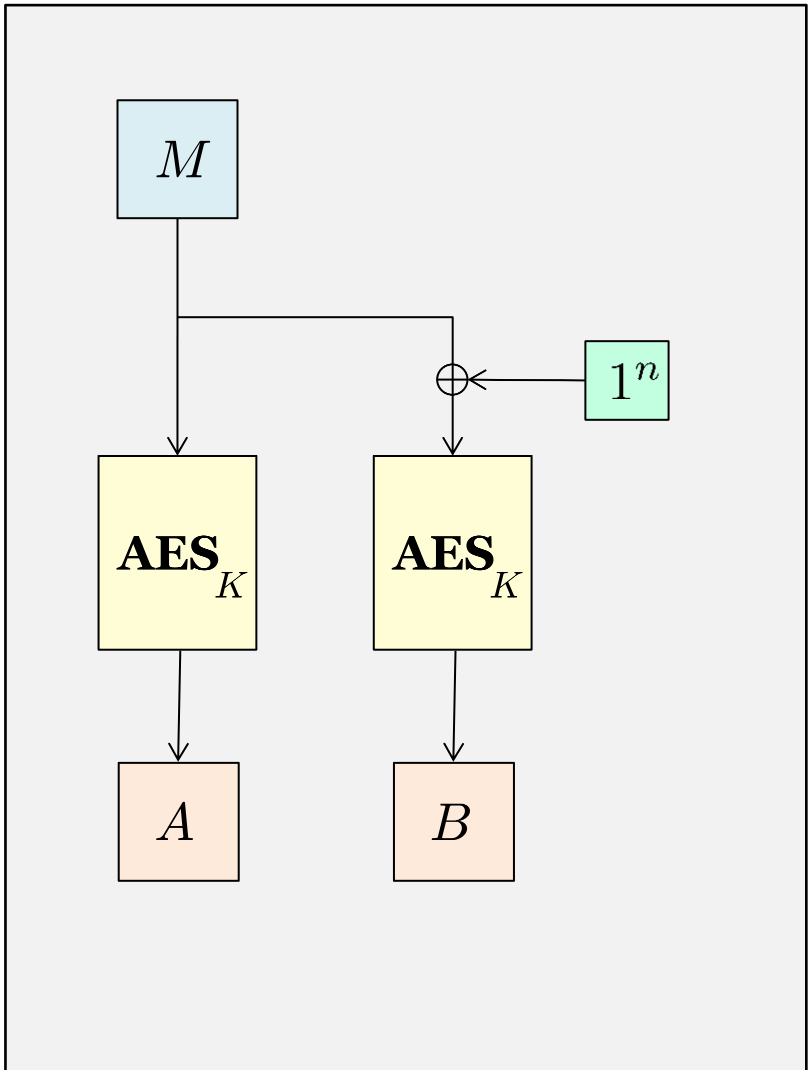
PRF Security

$\Downarrow$

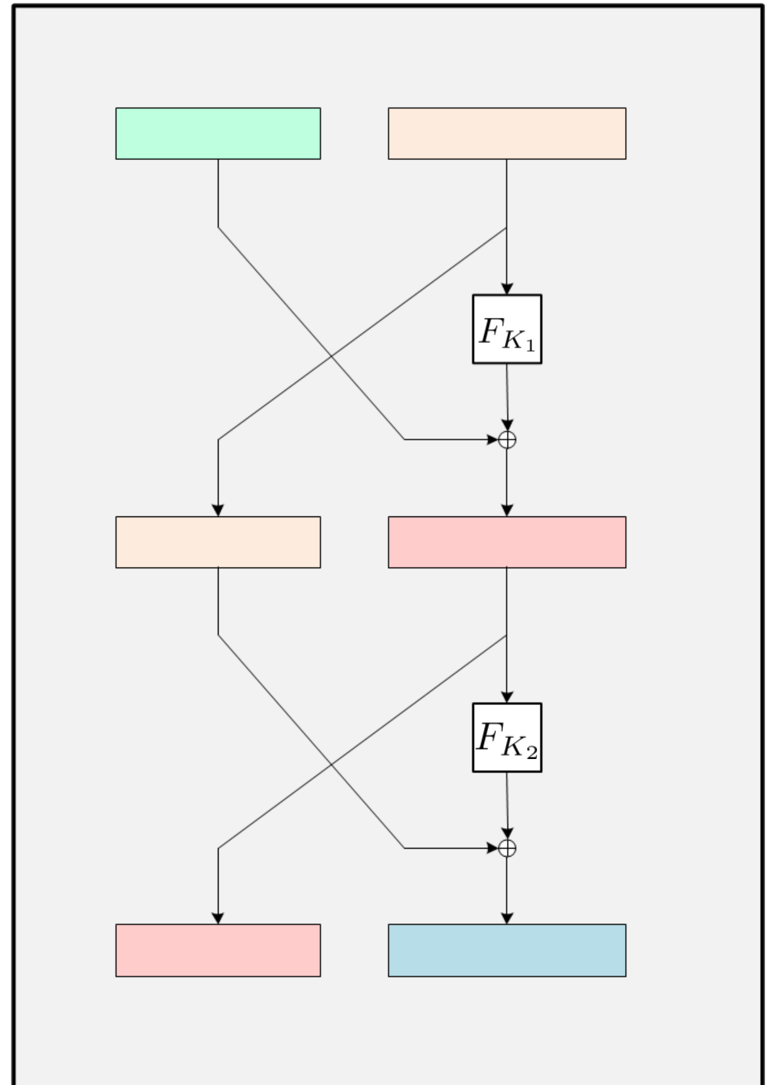
Key Recovery Security

Practice: PRF Attacks

$$E_K(M) = \mathbf{AES}_K(M) \| \mathbf{AES}_K(\overline{M})$$



Two-round Feistel

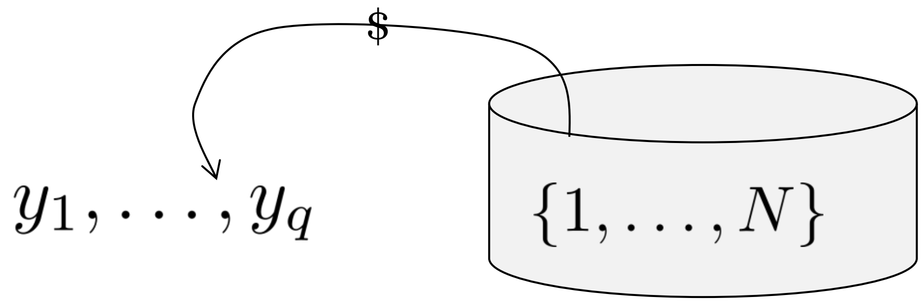


Agenda

1. Defining PRF Security

2. Birthday Attack

Birthday Problem

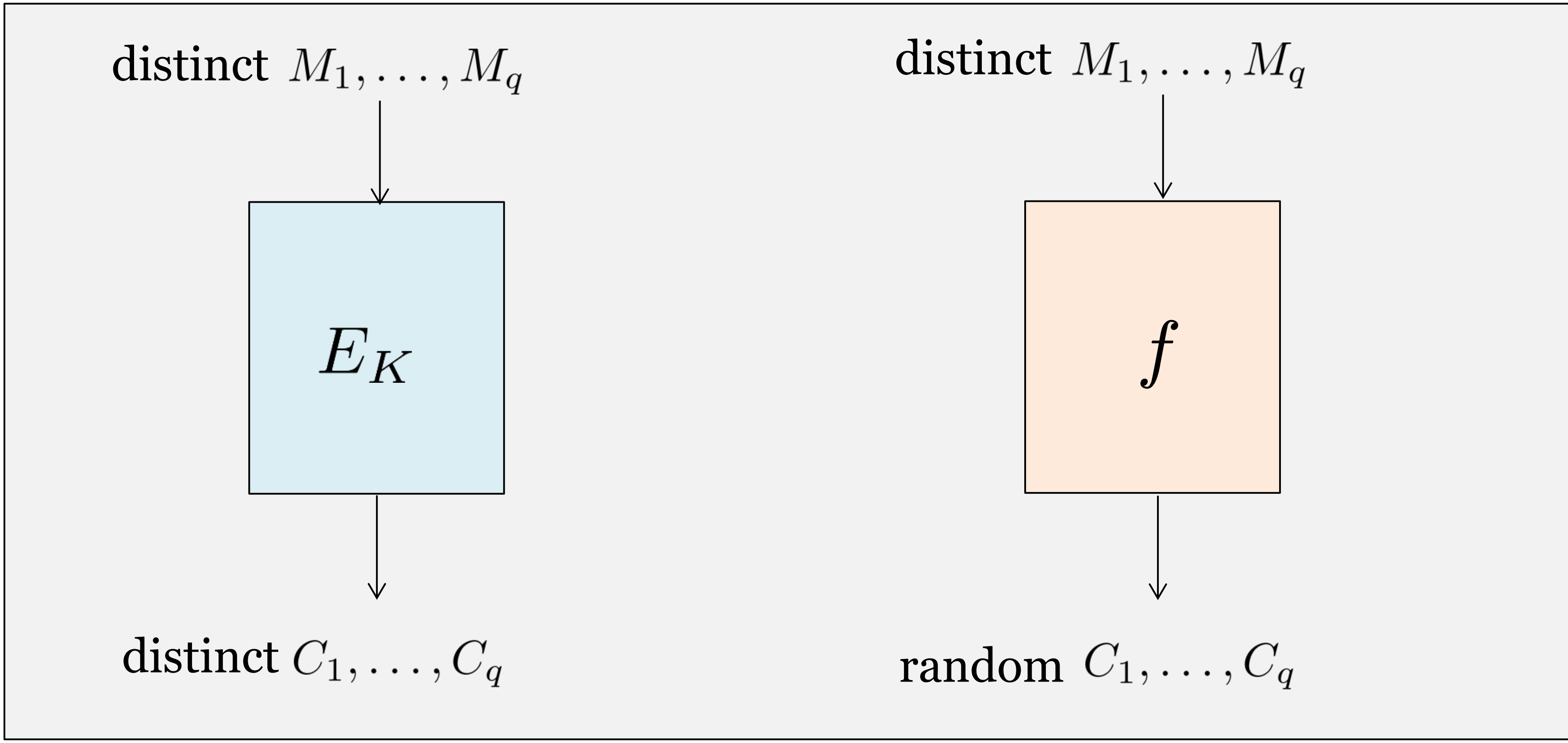


$$C(N, q) = \Pr[y_1, \ldots, y_q \text{ not distinct}]$$

Fact: For $q \leq \sqrt{2N}$,

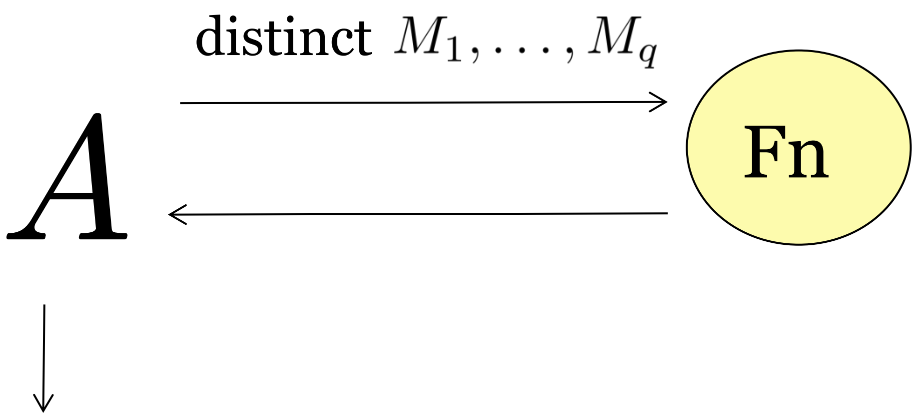
$$\frac{q(q-1)}{4N} \leq C(N, q) \leq \frac{q(q-1)}{2N}$$

Birthday Attack on PRF Security



Birthday Attack on PRF Security

$$E : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$$



Output 1 if $C_1, \ldots, C_q$ are distinct

$$\boxed{\mathbf{Adv}^{\mathrm{prf}}_{E}(A) = C(2^n, q) \approx \frac{q^2}{2^n}}$$

Need $2^{n/2}$ queries to break PRF security

Blockcipher	n	$2^{n/2}$	**Status**
DES, 2DES, 3DES	64	2^{32}	Insecure
AES	128	2^{64}	Secure

Does It Matter In Practice?

Sweet32: Birthday Attacks on 64-bit Blockciphers in TLS and OpenVPN [Bhargavan, Leurent 16]

HTTPS encryption via 3DES

Recover cookie after capturing 785GB